THE ICOSAHEDRAL SNOWFLAKE? (Part I)

Marjorie Senechal, Smith College
Mathematical Crystallography Workshop
Manila, May 24, 2017
1860 - Born in Edinburgh
1878 - Attended Edinburgh University to study Medicine
1880 - Moved to Cambridge University
1883 - BA Degree from Cambridge in Natural Science
1884 - Appointed Professor of Biology at University College Dundee
1896 - First expedition to the Bering Straits
1917 - Publication of *On Growth and Form*
1917 - Appointment to Chair of Natural History, St Andrews University
1937 - Awarded Knighthood
1948 - Died at home in St Andrews
Why look for more than physics and chemistry and mathematics, when these sciences have so much to tell us?
Mathematical Crystallography is a field in D’Arcy’s spirit.

Math has a lot to tell us about crystals.

Crystal form for sure (external shape, internal pattern) and crystal growth too.
Johannes Kepler
*De Nive Sexangula*, 1611
De Nive Sexangula is famous for “Kepler’s Conjecture,” which (in modern language) asserts that the fcc and hcc lattice sphere packings are the densest packings of infinitely many unit spheres in three-dimensional Euclidean space.

Kepler’s conjecture is notoriously difficult. Posed in 1611, it was finally proved by Thomas Hales in 2005.
Twelve unit spheres touching a central one.

- cubic close packing
- hexagonal close packing
But it’s also the first attempt to explain the growth and form of a crystal
A trapdoor in the ceiling opens and hundreds of identical ping-pong balls fall into a slightly slanted bin on the floor. There, as the balls flow downward, they spontaneously assemble in close-packed arrays. This symmetry disappears as they near the bin’s exit, where they tumble into a dumbwaiter and are whooshed to the ceiling. Then they are let fall again....
By “icosahedral snowflake”

I don’t mean a H$_2$O snowflake in the shape of an icosahedron

I do mean an analogue of Kepler’s snowflake model for icosahedral quasicrystals

By “icosahedral quasicrystal”

I don’t mean a 3-d tiling with icosahedral symmetry

I do mean a real crystal with icosahedral symmetry
By “crystal” I mean a solid-state material that satisfies the (relatively) new IUCr definition:

its diffraction pattern has bright sharp spots.

In more mathematical language: its diffraction measure has a nontrivial pure point component.

Crystals can be periodic or nonperiodic (aperiodic)

“Quasicrystals” are nonperiodic crystals whose diffraction patterns have symmetries incompatible with a periodic lattice.
50 years after Kepler:

Had I time and opportunity, I could make probable, that all these regular Figures that are so conspicuously *various* and *curious*, and do so adorn and beautifie such multitudes of bodies, arise onely from three or four several positions or postures of *Globular* particles, and those the most plain, obvious, and necessary conjunctions of such figur'd particles that are possible.

Robert Hooke, *Micrographia*, 1664

And then, 150 years after Hooke,
A rose is a rose is a rose.

A pyrite crystal is a . . .

Classify by what criterion?
Shape?
Chemistry?
Mode of formation?

The Abbé Haüy, 1743 - 1822
Haüy, 1810

fluorite

pyrite
Nucleation?

Growth: layer by layer

Form: finishing off the stack
Haüy had answered an important question:

**DOES THE PYRITOHEDRON “WANT TO BE” REGULAR?**
The centers of Hauy’s building blocks form a lattice.

Each crystal form is a lattice polytope and its faces are lattice polygons.

Is the regular dodecahedron a lattice polygon?
Is there a lattice to which these five points belong?

Also a lattice point

Solve for x:

\[ 1 + x = x \]
\[ -1 \quad 1-x \]

x is the golden ratio, which is irrational! So no such lattice exists.
AND SO THE LATTICE PARADIGM CONQUERED THE CRYSTAL KINGDOM
THEOREM (THE CRYSTALLOGRAPHIC RESTRICTION):
2-d and 3-d LATTICES CAN (ONLY) HAVE ROTATIONAL SYMMETRY OF ORDERS 2, 3, 4, AND 6
Auguste Bravais enumerated the 14 3-D point lattices in 1848.

1912: THE LATTICE PARADIGM WAS SOLIDIFIED BY THE DISCOVERY OF X-RAY DIFFRACTION
"THE CRYSTALLOGRAPHIC RESTRICTION"

impossible crystal symmetries!

was taught to science students for > 150 years
In 1982 Dan Shechtman discovered real crystals with icosahedral symmetry.

Figure 1. Daniel Shechtman’s diffraction pattern was tenfold; turning the picture a tenth of a full circle (36 degrees) results in the same pattern.
SURPRISE!!

THE CRYSTALLOGRAPHIC RESTRICTION ISN'T ABOUT CRYSTALS,

IT'S JUST A THEOREM ABOUT LATTICES!
If the atoms in a crystal don’t repeat periodically, how do they repeat?

PCMI entry in Park City’s July 4, 2014 Parade
Towards fivefold symmetry?

Crystallography is in for a minor upheaval, with the recognition of forbidden icosahedral symmetry by both construction and experiment.

Cold water on icosahedral symmetry

Lixus Pauling has produced an alternative explanation of the observation that solid manganese-aluminm alloy may have 5-fold symmetry on the atomic scale. How can the two views be reconciled?

Puzzling Crystals Plunge Scientists Into Uncertainty

MOST solid things are made of crystals, and for nearly two centuries scientists assumed that every crystal must have an orderly structure, its constituent atoms fixed at predetermined, periodic positions within a lattice framework. But the discovery of a new type of crystal that violates some of the accepted rules has touched off an explosion of conjecture and research that may lead to the founding of a new branch of science.

The finding has galvanized microstructure analysts, mathematicians, chemists, metallurgists and physicists in at least eight countries. According to one enthusiast, scientists around the world are now predicting a paper a day relating to the discovery, and an end to this torrent of research is nowhere in sight.

Whether this discovery will have practical consequences remains to be seen. But as one investigator put it: “If this kind of crystal proves to have properties as popular as its structure, the stuff seems certain to find important uses. That’s what one would expect in the field of condensed-matter physics.”

Skepticism Overthrown by Experimental Evidence

Among the many past achievements of condensed-matter physicists is the discovery of semiconducting crystals, which provided the
This mixture of theorizing and observation has now been put on a broad foundation by D. Levine and P. J. Steinhardt (Phys. Rev. Lett. 55, 2477; 1985) in a summary of largely unpublished theoretical investigations. The objective is to provide a framework for discussing what they call "quasi-crystals".

Fig. 4. A circle has been placed on each quasi-lattice point of the two-dimensional pattern to model a possible atomic structure.

Fig. 5. The central portion of the pattern of Fig. 4. The central structure view is the crystal core wave modulation of the surface. The pattern itself exhibits localized centers and repeats the shape of the quasi-crystal cells which gives rise to A.
Quasicrystals have “fundamentally altered how chemists conceive of solid matter.”
With the discovery of x-ray diffraction, form (atomic patterns) had pushed growth aside

but icosahedral crystals raised the question again.
Other speakers at this workshop have shown how the Penrose tiles can be generated by substitution (building supertiles) and by projection from higher-dimensional spaces.

But neither substitution nor projection describe growth in a physically reasonable way.

But what about matching rules? Does building Penrose tilings by matching rules model crystal growth?
Penrose tiles (also discovered by Robert Ammann) are two shapes: thick and thin rhombuses with notched edges. Only these eight arrangements can be expanded.

The notches don't let you fit two thick (or thin) rhombs together in parallel position. And that's the whole point: unlike wallpaper pattern, Penrose tilings are not predictable.

Color the shapes to find the patterns!

Pine cones, and crystals, googie these classified items to learn more about them and their role in small shells.

Fitting tiles together, you can extend the pattern shown here in all directions (and in infinitely many different ways), you'll see the same arrangements again and again, but the pattern will never exactly repeat. The same... but not the same.

Pentrose puzzles: the golden ratio

So are the Fibonacci numbers: 1, 2, 3, 5, 8, 13...

\[(x+1)/x = \phi \approx 1.6180339887\]

Is hidden in every Pentrose tiling.
Penrose tiles (two rhombs, upper left) and the eight vertex configurations permitted by their matching rules (shown here as notches)
Penrose tilings are not jig-saw puzzles. The matching rules are not local (though they are locally expressed). You can follow the rules yet reach dead ends. Can this patch be completed?
TO MODEL CRYSTAL GROWTH

WE GO BACK TO BASICS

. . . . AND THE ZOMETOOL LAB
Johannes Kepler

_De Nive Sexangula_, 1611
SPHERE-PACKINGS IN 3-D RAISE DIFFICULT PROBLEMS IN DISCRETE GEOMETRY:

KEPLER’S CONJECTURE

NEWTON’S QUERY

FRANK’S ERROR

FEJES-TOTH CONJECTURE
NEWTON WONDERED

how many spheres (of equal radius) can be packed around another in $R^3$?

$n = 1$: 2 (intervals)

$n = 2$: 6 (circles)

$n = 3$ ????
The questions was resolved (it’s 12) in 1953. Why did this take 300 years?
Surface area $S$ of sphere (radius 1) = $4\pi$.

Surface area $A$ “occupied” by an equal sphere touching it = $\pi(2 - \sqrt{3})$, and $2 - \sqrt{3} \approx 0.26794919243$.

Which means there is “room for” 13 -- and even 14 -- spheres to touch the surface (but not for 15).

13$(2 - \sqrt{3}) \approx 3.488$
14$(2 - \sqrt{3}) \approx 3.751$
15$(2 - \sqrt{3}) \approx 4.019$

So $14 \; A < S < 15 \; A$
“It was conjectured by David Gregory that a sphere can touch thirteen non-overlapping spheres equal to it.

"In a recent paper, proofs have been given that this in fact is impossible.

"In the present paper I outline an independent proof of this impossibility, certain details which are tedious rather than difficult being omitted."

John Leech’s two-page proof that 12 is max in $\mathbb{R}^3$

“We consider the configuration of the points of contact of the sphere with those around it . . . . we prove that there cannot be thirteen points on a sphere such that the (great circle) distance between any two of them is at least $\pi/3$.”
“Let us consider the network on the sphere formed by joining every pair of the points of the configuration whose distance apart is less than $d = \cos^{-1}(1/7)$.

[Note: $d \sim 81.7868^\circ$, whence $\pi/3 < d < \pi/2$]

“No two joins of this network cross, since any four points of the network form a quadrilateral of sides at least $\pi/3$ whose diagonals cannot both be less than $\pi/2$, the extreme case being that of the regular quadrilateral of side $\pi/3$ whose diagonals are both $\pi/2$.”
We may assume that the network is connected, since if it is in detached parts, these may be moved so as to bring vertices of each part within $d$ of another, and that no point of the configuration is within $d$ of only one other point, since any such point can be moved until it is within $d$ of another.

**LEMMA:** at most five joins of the network meet in any one vertex.

Because: Every angle of these polygons exceeds $\pi/3$, the lower bound, unattained in the network, being for the spherical triangle of sides $\pi/3$, $\pi/3$, $d$, whose angles are $\pi/3$, $\pi/3$, $\cos^{-1}(-1/7)$. 
The proof consists of showing by consideration of areas that for such a network to exist having thirteen vertices, it must divide the sphere into triangles except possibly for one quadrilateral, and that for topological reasons such networks do not exist.
By Euler's theorem on polyhedra, \( V + F = E + 2 \), we have

\[
2V - 4 = 2E - 2F
\]

\[
= 3F_3 + 4F_4 + 5F_5 + \ldots - 2(F_3 + F_4 + F_5 + \ldots)
\]

\[
= F_3 + 2F_4 + 3F_5 + \ldots
\]
He calculates the least areas of polygons of side $\pi/3$ drawn on $S$:

$$A\text{ (eq. Triangle)} = 3 \cos^{-1}(1/3) - \pi \sim 0.5513$$

$$A\text{ (Q)} = 2(d - \pi/3) \sim 1.334$$

$$A\text{ (eq. Pentagon)} \sim 2.226$$

Note that $A\text{(Q)} \geq 2A\text{(T)}$ and $A\text{(P)} \geq 3A\text{(T)}$.

He shows that, more generally, $A\text{(n-gon)} - (n-2)A\text{(T)}$ increases with $n$.

(Q is a quadrilateral whose shortest diagonal is $d$)
The area of the network is thus

\[ \geq 0.5513 \, F_3 + 1.334 \, F_4 + 2.226 \, F_5 + \ldots \]

\[ = 0.5513 \, (F_3 + 2F_4 + 3F_5 + \ldots) + 0.231F_4 + 0.0572(F_5 + \ldots) \]

\[ = 0.5513 \, (2V - 4) + 0.231F_4 + 0.572 \, (F_5 + \ldots) \]

Now the area of the sphere is \(4\pi\), whence

\[ 2 \, V - 4 \leq 4\pi / 0.5513 = 22.8, \]

so \( V \leq 13. \)
Suppose now, if possible, that $V = 13$.

Then $2V - 4 = 22$, and we have

$$4\pi \geq 0.5513 \times 22 + 0.231 \ F_4 + 0.572(\ F_5 + ...),$$

i.e. $0.438 \geq 0.231F_4 + 0.572 \ (F_5 + ...)$.

Thus $F_4 = 0$ or $1$ and $F_5 = ... = 0$,

and the network has to divide the sphere into triangles except possibly for the one quadrilateral.
Case 1: \( F_4 = 0 \).

Then all the faces are triangles. Then \( E = 33 \) and the average number of edges per vertex is \( > 5 \). But we have shown at most 5 edges can meet at any vertex.

Case 2. \( F_4 = 1 \).

Then Euler's theorem gives \( F_3 = 20, E = 32 \), and 4 joins meet at some one vertex and 5 at each other.

BUT despite the consistency of these numbers, there is in fact no polyhedron which has them. I know of no better proof of this than sheer trial. **QED**
SO, AT MOST 12 SPHERES OF RADIUS R CAN PACK AROUND ANOTHER SPHERE OF THAT RADIUS.

BUT BECAUSE THERE IS WIGGLE ROOM, THERE ARE INFINITELY MANY WAYS TO DO IT.
most symmetrical packing of 12 spheres around a central sphere
densest packing of 12 spheres around a central sphere

Which is the best model for our purposes? And why?
The Icosahedral Snowflake?

Part II

Joint work with Pablo Damasceno, Yoav Kallus, Jean Taylor, and Erin Teich
COMPARE AND CONTRAST:

most symmetrical arrangement of 12 spheres around a central sphere

densest packing of 12 spheres around a central sphere
The outer spheres touch only the central one, not each other.
Which packing of 13 spheres

• Maximizes the average number of contacts per sphere?
• Minimizes the sum of the inverses of their pairwise distances\(^2\) (the Thomson problem)?
• Maximizes the least distance between any pair (the Tammes problem)?
• Minimizes the Voronoi cell of the central sphere (Fejes-Toth problem)?

These are separate problems – the “bests” don’t always match up.
How do you optimally arrange $N$ repulsive points on a sphere? Just over 100 years ago, Thomson considered this very problem in an attempt to explain the periodic table in terms of rigid electron shells, the "plum pudding" model of the atom. The problem has resurfaced in many fields including multielectron bubbles in superfluid helium, virus morphology, protein s-layers, coding theory, is equivalent to many other problems in biology, math, physics, and computer science, and can be applied to such problems as structural chemistry, the design of multibeam laser implosion devices, and the optimum placement of communication satellites.
Tammes’ problem

“On the origin of number and arrangement of the places of exit on pollen grains” – P. M. L. Tammes, Dissertation, Groningen, 1930

Solved so far for $3 \leq N \leq 13$ and $N = 24$
<table>
<thead>
<tr>
<th>N</th>
<th>Tammes</th>
<th>Thomson</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>poles of a sphere</td>
<td>same</td>
</tr>
<tr>
<td>3</td>
<td>lie on great circle</td>
<td>same</td>
</tr>
<tr>
<td>4</td>
<td>regular tetrahedron</td>
<td>same</td>
</tr>
<tr>
<td>5</td>
<td>bi-pyramid or square pyramid</td>
<td>bi-pyramid</td>
</tr>
<tr>
<td>6</td>
<td>regular octahedron</td>
<td>same</td>
</tr>
<tr>
<td>7</td>
<td>hexagonal pyramid</td>
<td>pentagonal bipyramid</td>
</tr>
<tr>
<td>8</td>
<td>cubic antiprism <em>(not the cube!)</em></td>
<td>elongated cubic antiprism</td>
</tr>
</tbody>
</table>

**N = 12:** same: the regular icosahedron, for both of these criteria and for many others
Plato: water particles are icosahedra
“I infer that this [the icosahedral arrangement of spheres] will be a very common grouping in liquids, that most of the groups of twelve atoms around one will be in this form, that freezing involves a substantial rearrangement, and not merely an extension of the same kind of order from short distances to long ones . . .”

2003: Ken Kelton et al proved (experimentally) that Plato and Frank were right about water particles.

His experiments also challenged received ideas on how crystals start to grow.

Physics Today, 56 (7) 24 2003
“Understanding the glassy state — a solid-like state in which the atoms are in irregular positions much like atoms in a liquid — is one of the great unsolved mysteries in condensed-matter physics.

“Physicists believe that glass is formed when a liquid is cooled and its constituent atoms are unable to arrange themselves in a stable crystalline state. Instead, the atoms become trapped in a state of ‘dynamical arrest.’ ”
The researchers watched as a colloidal liquid cooled to become a gel and saw the formation of structures with five-fold rotational symmetry — a hallmark of icosahedral structures.

Using computer analysis, they found that these structures became more numerous upon cooling to form jammed icosahedra-like structures.
In the system under investigation, which forms a dodecagonal quasicrystal, we show that this process occurs through the assimilation of stable icosahedral clusters by the growing quasicrystal. Our results demonstrate how local atomic interactions give rise to the long-range aperiodicity of quasicrystals.
We observe that the system consistently forms a QC for all seed sizes, positions, and approximant structures, indicating that the system does not copy the seed, but rather incorporates atoms into the solid via a different paradigm.
DOES THE ICOSAHEDRON "GROW" INTO A PERIODIC CRYSTAL STRUCTURE?
Can these $n = 12$ clusters be transformed smoothly into each other (without breaking contact with the center ball)?

**NO!** (F.C. Frank, 1952)

**YES!** R. Kusner, W. Kusner, J. Lagarias and S. Schlosman, 2017
top three: slide toward north pole

central six: rotate and flatten

bottom three: slide toward south pole

But does this transformation occur spontaneously in real materials? Probably not.
Q. Which nanocluster wins on the energy scale?

A. For clusters of atoms of Cu, Pd, Ag, or Ni of size $n$, the icosahedron beats fcc for $18 < n < 2000$. 
Equilibrium structures of copper clusters up to 10,000 atoms are studied. Icosahedral closed-shell clusters are most stable up to ~2500 atoms and the Wulff polyhedra are favored for larger clusters.

Cuboctahedral closed-shell clusters up to ~2000 atoms are unstable. They undergo a nondiffusive transition to an icosahedral structure at low temperatures.

A computer-generated Mackay-type icosahedron with seven layers,

FCC (Face Centered Cubic)
Crystal Structure

Alan Mackay
THE ICOSAHEDRON: STEM CELL OF THE SOLID STATE?

The atoms of many intermetallic periodic crystals have 1-, 2- or 3-shell coordination polyhedra with icosahedral symmetry.
Key insight (Chris Henley): some periodic intermetallic crystals approximate aperiodic crystals, much as rational numbers approximate irrationals.

In each case, the same local configurations appear in both.

EXERCISE: WHY WEREN’T ICOSAHEDRAL QUASICRYSTALS DISCOVERED EARLIER?
Simulated single-component icosahedral crystal

Atomic clusters frequently found in intermetallics (periodic and aperiodic)

Bergman cluster

Mackay cluster

Tsai cluster (Yb-Cd type)

(Courtesy Marc de Boisseau and C. Pay Gomez)
Stable clusters in quasicrystals - fact or fiction?

Walter Steurer, 2005

What should be done:

• define the term cluster geometrically, chemically and physically;

• describe its structure and properties in quasicrystals;

• describe its role for the formation and stabilization of quasicrystals.
Eclectic reading (in the recent quasicrystal literature) and random listening (to quasicrystallographers) suggest that the words “tiling”, “cluster”, “packing”, and “covering” are used intuitively.

And when authors do define their terms, these definitions may differ from author to author (or even paper to paper)

Example: rigid cluster

*Arkus et al:* a configuration of N unit spheres that maximizes the number of contacts between them

*Holmes-Cerfon:* ditto that cannot be deformed continuously by any finite amount and still maintain all contacts.
YbCd: THE FIRST IQC STRUCTURE TO BE SOLVED
Figure 1 from Takakura et al. 2007. Cluster based description; clusters can overlap.

Tsai clusters can form approximant periodic crystals.

Tsai clusters meet face-to-face (b-linkage) or overlap in an OR (c-linkage).
Jean Taylor with her ZomeTool model of the Tsai cluster, Park City, Utah, July 2014
“An exceptional dynamical flexibility”

M. de Boisseau,
keynote lecture,
International Union of Crystallography Congress,
Montreal
August 2014
Soft packings, nested clusters, and condensed matter

American Institute of Mathematics, San Jose, California
19-23 September, 2016

Organizers: Karoly Bezdek, Nikolai Dolbilin, Egon Schulte, and Marjorie Senechal

This workshop, sponsored by AIM and the NSF, will be devoted to modeling the geometry of condensed matter. The workshop will focus on "soft packing" and "nested clustering" phenomena in discrete geometric structures and their applications to understanding the internal atomic structure of solids and fluids. In particular, the workshop seeks to integrate the theories of tilings, coverings, and packings, and to develop new discrete geometric concepts and tools needed to study aperiodic structures such as aperiodic crystals. For more details visit the CCDG page of the workshop and the workshop website.

The main problems for the workshop include:

- Nested clustering in aperiodic structures in Euclidean space. Structure theory for nested clusters; local rules for building global structures; local characterizations of global structures; and creating a catalogue of nested clusters.
- Soft sphere packing and its relationship with classical sphere packing. Optimal soft packings; density estimates; and soft packing analogues of the kissing number and contact number.
- Delaunay point sets in Euclidean space. Classification of Delaunay sets; geometric structures over Delaunay sets; local theorems; and Delaunay graphs.

These three problem areas are interrelated and arise in applications.
“What is the densest packing of congruent soft balls in Euclidean 3-space?”

ETC

Károly Bezdek,

Zsolt Langi
Tsai cluster (partially removed outer two shells in front to see inside)
TDI clusters

Tetrahedron
Dodecahedron
Icosahedron
HOW TDIs COULD MODEL GROWTH

The red shared OR appears naturally as the two Tsai clusters are filled in.
WHY MIGHT TDIs EXIST IN THE MELT?

A TDI, with Yb atoms replaced by Cd atoms, closely resembles the densest of the packings that Erin Teich had earlier computed for 36 identical balls.

In that configuration, 32 of these balls spontaneously form an outer shell touching the confining sphere (their centers forming a spherical code), with the remaining four inside.

Confinement to a sphere is a physically reasonable model, as the surrounding liquid produces on average a given pressure around the aggregate of 36 atoms, perhaps encouraging them to adopt this configuration.
These 32 balls lie approximately at the vertices of a regular dodecahedron and a dual regular icosahedron.

The centers of the four balls inside approximate the vertices of a regular tetrahedron, which has some “wiggle room.”

But in a real (YbCd) TDI, the dodecahedral and icosahedral shells of the TDI have different radii and their balls have different radii.
Summary of our AIM working group conjectures:

TDIs exist (temporarily) in the melt.

Growth is mainly by adding more TDIs via c-linkages. Then Cd atoms settle in between the TDIs to finish Tsai clusters. b-linkages result automatically.

Which crystal structure you get depends only on Cd:Yb.

Order becomes long-range, giving sharp spots with icosahedral symmetry in the diffraction pattern because the c-linkages and b-linkages lock the Tsai clusters into the same orientation, same angles, same distances.
WHAT ARE THE MATHEMATICAL CHALLENGES?

projection of the 6-cube
From Steurer again:

• define the term cluster geometrically, chemically and physically;

• describe its structure and properties in quasicrystals;

• describe its role for the formation and stabilization of quasicrystals.
More generally:

Develop this approach to YbCd crystals more fully and in collaboration with Tsai and his colleagues.

Are we on to something? Or is YbCd an exceptional case? Can this approach can be extended to other aperiodic crystals and their approximants?

Formulate the appropriate mathematical contexts and questions.

IS the icosahedron a stem cell of condensed matter?
what is the geometry of water? liquid crystals? the glassy state?
Sphere Packings and Stem Cells

Marjorie Senechal

Abstract: For Alan Mackay, on his 90th birthday, we recast his notion of crystalloids and crystallites as finite nano-sized sphere-packings and discuss current mathematical/computational research on their geometric structures. We suggest that icosahedral clusters are (metaphoric) stem cells for condensed matter.

Keywords: sphere-packing, nanocluster, crystallloid, crystallite, icosahedron, and Mackay.

The 6th Annual World Congress of Molecular Medicine 2017

Time: September 25-27, 2017

Venue: Xi'an, China

Website: http://www.bitcongress.com/molmed2017

Dear Dr. Marjorie Senechal,

On behalf of the organizing committee, we are pleased to announce that the 6th Annual World Congress of Molecular Medicine 2017 (MolMed-2017) will be held on Sep 25-27, 2017 in Xi’an, China. This year’s theme is Open A New Era of Human Health. It is an honor to welcome you to attend this meeting and present a speech on Sphere packings as stem cells... at MolMed-2017.
While I have thought to shew the naturalist how a few mathematical concepts and dynamical principles may help and guide him, I have tried to shew the mathematician a field for his labor – a field which few have entered and no man has explored.

D’Arcy W. Thompson, *On Growth and Form*, 1917