International Autumn School on Fundamental and Electron Crystallography (IASFEC)
8-13 October 2017, Sofia, Bulgaria

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Notation (reminder)

$uvw$: coordinates of a lattice nodes in direct space

$[uvw]$: direction indices in direct space

$\langleuvw\rangle$: set of equivalent (symmetry-related) directions in direct space

$(hkl)$: Miller indices of a lattice plane in direct space

$\{hkl\}$: crystal form (set of crystal faces equivalent by symmetry). By extension, also set of equivalent (symmetry-related) lattice planes in direct space

$hkl$: coordinates of a lattice node in reciprocal space; also, Laue indices of a diffraction

$[hkl]^*$: direction indices in reciprocal space

$\langle hkl \rangle^*$: set of equivalent directions in reciprocal space

$(hkl)^*$: Miller indices of a lattice plane in reciprocal space

$\{hkl\}^*$: set of equivalent lattice planes in reciprocal space
A zone axis is a lattice row parallel to the intersection of two (or more) families of lattice planes. It is denoted by \([u \, v \, w]\). A zone axis \([u \, v \, w]\) is parallel to a family of lattice planes of Miller indices \((hkl)\) if (Weiss law):

\[uh + vk + wl = 0\]

The indices of the zone axis defined by two lattice planes \((h_1,k_1,l_1)\), \((h_2,k_2,l_2)\) are given by:

\[
\frac{u}{k_1 \, l_1} = \frac{v}{l_2 \, h_2} = \frac{w}{k_2 \, h_2}
\]

Conversely, any crystal face can be determined if one knows two zone axes parallel to it (zone law)

Three lattice planes have a common zone axis (are in zone) if their Miller indices \((h_1,k_1,l_1)\), \((h_2,k_2,l_2)\), \((h_3,k_3,l_3)\) satisfy the relation:

\[
\begin{vmatrix}
h_1 & k_1 & l_1 \\
h_2 & k_2 & l_2 \\
h_3 & k_3 & l_3
\end{vmatrix} = 0
\]
Compute the zone axis

Let the Miller indices of two lattice planes be \((h_1,k_1,l_1), (h_2,k_2,l_2)\).

<table>
<thead>
<tr>
<th>(h_1)</th>
<th>(k_1)</th>
<th>(l_1)</th>
<th>(h_2)</th>
<th>(k_2)</th>
<th>(l_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(u)</td>
<td>(v)</td>
<td>(w)</td>
</tr>
</tbody>
</table>

Remove common factor, if any

Exercise: zone axis for faces (001) and (101)

\[
\begin{array}{c|ccccc|c}
0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 1 \\
\hline
0 & 1 & 0 & 0 \\
\end{array}
\]

Exercise: zone axis for faces (231) and (362)

\[
\begin{array}{c|cccccc|c}
2 & 3 & 1 & 2 & 3 & 1 \\
3 & 6 & 2 & 3 & 6 & 2 \\
\hline
0 & 1 & 3 \\
\end{array}
\]

Low-index zone axes correspond to lattice rows parallel to several lattice planes

\[
\begin{array}{c|ccccc|c}
h_1 & 0 & l_1 & h_1 & 0 & l_1 \\
\hline
h_2 & 0 & l_2 & h_2 & 0 & l_2 \\
\hline
0 & n & 0 \\
\end{array}
\]

\[n = l_1 h_2 - l_2 h_1\]
A **Zone-Axis Pattern** (ZAP) is observed when a high symmetry \([uvw]\) direction of the crystal is parallel to the incident beam. The diffraction spots on the pattern are arranged along **Laue zones**.
Zone-Axis Pattern obtained along the \([uvw]\) zone axis. The diffraction pattern shows the \((uvw)^*\) reciprocal lattice plane. This pattern is produced from direct lattice planes in the \([uvw]\) zone, \textit{i.e.} “vertical” in the microscope and (almost) parallel to the electron beam. If the Laue indices are known, then the indices of the zone axis are easily obtained.
Exercise

Find the zone axis from the following 2D diffraction pattern (no intensity shown)
How to choose a suitable zone axis for EM?

Zone axes \([uvw]\) of interest in EM correspond to lattice plane \((hkl)\) of small larger interplanar distance \(d(hkl)\), \(i.e.\) of high reticular density, which means small Miller indices.

\[
d(hkl) = 1/||\mathbf{r}^*(hkl)|| = (hkl|\mathbf{G}^*|hkl)^{1/2}
\]
The perpendicularity between a lattice direction \([uvw]\) and a lattice plane \((hkl)\) depends on the symmetry of the lattice – which on its turn depends on the presence or absence of metric specialisation.

<table>
<thead>
<tr>
<th>Lattice symmetry</th>
<th>Lattice plane</th>
<th>Lattice direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{1})</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2(/m) ((b)-unique)</td>
<td>(010)</td>
<td>[010]</td>
</tr>
<tr>
<td>2(/m) 2(/m) 2(/m)</td>
<td>(100)</td>
<td>[100]</td>
</tr>
<tr>
<td></td>
<td>(010)</td>
<td>[010]</td>
</tr>
<tr>
<td></td>
<td>(001)</td>
<td>[001]</td>
</tr>
<tr>
<td>4(/m) 2(/m) 2(/m)</td>
<td>(001)</td>
<td>[001]</td>
</tr>
<tr>
<td></td>
<td>(hkl)</td>
<td>([hkl])</td>
</tr>
<tr>
<td>(\bar{3}) 2(/m) and 6(/m) 2(/m) 2(/m) (hexagonal axes)</td>
<td>(001)</td>
<td>[001]</td>
</tr>
<tr>
<td></td>
<td>(hki)</td>
<td>([2h+k,h+2k,0])</td>
</tr>
<tr>
<td>4(/m) (\bar{3}) 2(/m)</td>
<td>(hkl)</td>
<td>([hkl])</td>
</tr>
</tbody>
</table>